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PROTECTIVE PROPERTIES OF HOT-AIR SCREENS

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An analytical and numerical study of protective properties of hot-air screens is performed for a plane wave with a triangular profile. The reduction of the shock-wave pressure behind the screen is only observed at temperatures higher a certain critical value; at lower temperatures, the pressure is higher than that in a shock wave traveling in an isothermal gas. With allowance for real properties of air, an analytical relation between the critical temperature of the hot screen and the incident-wave intensity is obtained.

Prosser [1] suggested a method of protection against thermonuclear-explosion shock waves (SW), which implies that an extended layer of radiation-absorbing particles is artificially formed at a certain height above the ground level immediately before the explosion. The leading light radiation of the nuclear explosion heats the particles to form a hot-air screen. The SW, which arrive later, interacts with this layer and becomes weaker; its destructive action on nearby terrestrial objects also decreases.

The pressure decrease at the SW front behind the hot-air screen can be understood within the acoustic approximation (for a low-intensity plane wave). This approach yields the following expression for the maximum excess pressure at the front of the SW behind the screen [2]:

$$\Delta p_{m,2} = \Delta p_{m,0}\, 4\rho_0 c_0 \rho_1 c_1/(\rho_0 c_0 + \rho_1 c_1)^2. \tag{1}$$

Here ρ is the gas density and c is the velocity of sound in the gas; the subscripts 0, 1, and 2 refer to the media in front of (medium 1), inside (medium 2), and behind the hot-gas screen (medium 3).

For identical pressures in the screen and in the ambient atmosphere, we have $\rho c \sim 1/\sqrt{T}$, where T is the absolute gas temperature; therefore, relation (1) can be rewritten as

$$\Delta p_{m,2}/\Delta p_{m,0} = 4\sqrt{T_1/T_0}/(1 + \sqrt{T_1/T_0})^2. \tag{2}$$

For $T_1 \geqslant T_0$, dependence (2) is a function monotonically decreasing down from unity.

Andrushchenko et al. [3] and Belotserkovskii et al. [4] reported numerical solutions of a two-dimensional axisymmetric problem on reflection of an explosion-generated spherical SW from a flat surface after its interaction with a hot-air screen located at a certain distance from the surface. For an intense incident SW and a moderate temperature of the screen, an increase rather than a decrease in pressure is observed in the vicinity of the explosion epicenter, in contrast to the case without the hot-air screen. However, in these publications, it remained unclear whether this phenomenon was a consequence of the particular formulation of the problem or that of some specific properties of the heated gas layer itself.

Having numerically solved a one-dimensional problem about a plane wave traversing a heated region, we showed [5] that the wave intensity behind the screen increases irrespective of the pressure at the front of the incident SW if the screen temperature is low, which implies that the effect should be attributed to some specific properties of the air screen itself.

To analyze in more detail the phenomenon of interest for low-intensity shock waves, we use the nonlinear theory of short waves, which is applicable to waves of arbitrary wavelengths in the plane case. We consider an

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SW with a triangular profile, where the excess pressure diagram at a certain initial distance $x = x_0$ is set by the following dependence:

$$\Delta p_0^0 = \begin{cases} \Delta p_{m,0}^0(1 - t^0/\tau_0^0), & t^0 \leqslant \tau_0^0, \\ 0, & t^0 > \tau_0^0. \end{cases}$$

Here $\Delta p_{m,0}^0$ is the excess pressure at the SW front and τ_0^0 is the duration of the compression stage; the time t^0 is measured from the moment the wave arrives at the section $x = x_0$.

Using the general theory of short waves [6], one can easily obtain the relations for the maximum excess pressure at the SW front and for the duration of the compression stage in a plane wave traveling in a medium with an initial pressure p_0 and velocity of sound c_0:

$$\Delta p_{m,0} = \Delta p_{m,0}^0 \Big/ \sqrt{1 + \frac{k+1}{2k}\,\frac{\Delta p_{m,0}^0}{p_0}\,\frac{x}{\tau_0^0 c_0}}, \quad \tau_0 = \tau_0^0 \sqrt{1 + \frac{k+1}{2k}\,\frac{\Delta p_{m,0}^0}{p_0}\,\frac{x}{\tau_0^0 c_0}}. \tag{3}$$

Here k is the ratio of specific heats of the gas; the superscript 0 refers to the section $x = x_0$.

A comparison of the values predicted by Eqs. (3) and the numerical solution of the problem for air ($k = 1.4$) [5] shows that the errors in $\Delta p_{m,0}$ and τ_0 predicted by the theory of short waves for $\Delta p_{m,0}^0/p_0 = 0.15$ are less than 1.3 and 2.0%, respectively, and increase to 10 and 15% for $\Delta p_{m,0}^0/p_0 = 0.5$.

Let the section $x = x_1$ be a boundary of a hot-air screen with a temperature T_1, density ρ_1, and velocity of sound c_1 under the same pressure p_0. In this case, there occurs a discontinuity at the interface between the heated and cool regions, generating an SW that enters the screen with an initial pressure (in the acoustic approximation)

$$\Delta p_{m,1}^1 = \Delta p_{m,0}^1 \frac{2\rho_1 c_1}{\rho_0 c_0 + \rho_1 c_1} = \Delta p_{m,0}^0 \frac{2}{1+y} \Big/ \sqrt{1 + \frac{k+1}{2k}\,\frac{\Delta p_{m,0}^0}{p_0}\,\frac{x_1}{\tau_0^0 c_0}}; \tag{4}$$

the time required for the wave to traverse the section $x = x_1$ is

$$\tau_1^1 = \tau_0^1 = \tau_0^0 \sqrt{1 + \frac{k+1}{2k}\,\frac{\Delta p_{m,0}^0}{p_0}\,\frac{x_1}{\tau_0^0 c_0}}. \tag{5}$$

Here $y = \sqrt{T_1/T_0}$.

The variation of parameters of the wave propagating in medium 1 is described by Eqs. (3) with appropriate replacement of the constants and argument:

$$\Delta p_{m,1} = \Delta p_{m,1}^1 \Big/ \sqrt{1 + \frac{k+1}{2k}\,\frac{\Delta p_{m,1}^1}{p_0}\,\frac{x - x_1}{\tau_1^1 c_1}}, \quad \tau_1 = \tau_1^1 \sqrt{1 + \frac{k+1}{2k}\,\frac{\Delta p_{m,1}^1}{p_0}\,\frac{x - x_1}{\tau_1^1 c_1}}\,.$$

In view of (4) and (5), and with allowance for the condition $c_1/c_0 = y$, we have

$$\Delta p_{m,1} = \Delta p_{m,0}^1 \frac{2}{1+y} \Big/ \sqrt{1 + \frac{k+1}{2k}\,\frac{2}{y(1+y)}\,\frac{\Delta p_{m,0}^1}{p_0}\,\frac{x - x_1}{\tau_0^1 c_0}}\,, \qquad \tau_1 = \tau_0^1 \sqrt{1 + \frac{k+1}{2k}\,\frac{2}{y(1+y)}\,\frac{\Delta p_{m,0}^1}{p_0}\,\frac{x - x_1}{\tau_0^1 c_0}}\,. \tag{6}$$

In the wave that leaves the hot-gas screen at the section $x = x_2$ and enters medium 2 with a density ρ_0 and velocity of sound c_0, the initial parameters with allowance for (6) are

$$\Delta p_{m,2}^2 = \Delta p_{m,1}^2 \frac{2\rho_0 c_0}{\rho_0 c_0 + \rho_1 c_1} = \Delta p_{m,0}^1 \frac{4y}{(1+y)^2} \Big/ \sqrt{1 + \frac{k+1}{2k}\,\frac{2}{y(1+y)}\,\frac{\Delta p_{m,0}^1}{p_0}\,\frac{h}{\tau_0^1 c_0}}\,,$$

$$\tau_2^2 = \tau_1^2 = \tau_0^1 \sqrt{1 + \frac{k+1}{2k}\,\frac{2}{y(1+y)}\,\frac{\Delta p_{m,0}^1}{p_0}\,\frac{h}{\tau_0^1 c_0}}\,, \tag{7}$$

where $h = x_2 - x_1$ is the screen thickness.

The parameters of the wave propagating in a gas with a constant initial temperature T_0 from the section $x = x_1$ by the screen thickness h are given by the relations

$$\Delta p_{m,0}^2 = \Delta p_{m,0}^1 \Big/ \sqrt{1 + \frac{k+1}{2k}\,\frac{\Delta p_{m,0}^1}{p_0}\,\frac{h}{\tau_0^1 c_0}}\,, \quad \tau_0^2 = \tau_0^1 \sqrt{1 + \frac{k+1}{2k}\,\frac{\Delta p_{m,0}^1}{p_0}\,\frac{h}{\tau_0^1 c_0}}\,,$$

TABLE 1

A	y_*	T_*/T_0	$(y_*)_a$
0	1	1	1
0.1	1.268	1.608	1.251
0.5	2.071	4.289	2.029
1.0	2.854	8.145	2.703
2.0	4.105	16.851	3.596
4.0	6.040	36.482	4.491
10.0	10.127	102.556	6.096

TABLE 2

A	y_m	$\Delta p^2_{m,2}/\Delta p^2_{m,0}\big\|_m - 1,\%$
0	1	0
0.1	1.1254	0.402
0.5	1.4311	4.55
1.0	1.672	10.10
2.0	2.000	19.26
4.0	2.435	31.96
10.0	3.214	52.60

which are similar to relations (3). Then, the ratios of the excess pressure at the wave front and the duration of the compression stage in the wave leaving the screen to the respective parameters in the wave traveling in an isothermal gas with a temperature T_0 are

$$\frac{\Delta p^2_{m,2}}{\Delta p^2_{m,0}} = \frac{4y}{(1+y)^2}\sqrt{1+\frac{k+1}{2k}\,\frac{\Delta p^1_{m,0}}{p_0}\,\frac{h}{\tau^1_0 c_0}}\Bigg/\sqrt{1+\frac{2}{y(1+y)}\,\frac{k+1}{2k}\,\frac{\Delta p^1_{m,0}}{p_0}\,\frac{h}{\tau^1_0 c_0}}\,,$$

$$\frac{\tau^2_2}{\tau^2_0} = \sqrt{1+\frac{2}{y(1+y)}\,\frac{k+1}{2k}\,\frac{\Delta p^1_{m,0}}{p_0}\,\frac{h}{\tau^1_0 c_0}}\Bigg/\sqrt{1+\frac{k+1}{2k}\,\frac{\Delta p^1_{m,0}}{p_0}\,\frac{h}{\tau^1_0 c_0}}\,.$$

It follows from here that the change in the relative parameters of the wave behind the hot-air screen depends only on the dimensionless parameter $A = \dfrac{k+1}{2k}\,\dfrac{\Delta p^1_{m,0}}{p_0}\,\dfrac{h}{\tau^1_0 c_0}$, which is a quantity varying in proportion to the product of the dimensionless excess pressure at the front of the incident shock wave and the screen thickness normalized to the wavelength. Therefore, we finally obtain

$$\frac{\Delta p^2_{m,2}}{\Delta p^2_{m,0}} = \frac{4y}{(1+y)^2}\sqrt{\frac{1+A}{1+2A/(y(1+y))}}\,; \tag{8}$$

$$\frac{\tau^2_2}{\tau^2_0} = \sqrt{\frac{1+2A/(y(1+y))}{1+A}}\,. \tag{9}$$

As the temperature in the hot-gas screen increases from the initial value ($y \geqslant 1$), function (8) first increases from unity to a maximum value, then decreases, attaining unity at a certain temperature $y = y_*$, and tends to zero as $y \to \infty$. Thus, the excess pressure in the wave behind the screen can decrease if the screen temperature is higher than a certain critical value ($T_1 > T_*$), irrespective of the incident-wave intensity and screen thickness ($A > 0$). The time required for the wave to traverse the screen ($y > 1$) given by formula (9) is always smaller than the time required for a wave with identical parameters to traverse the same length in an isothermal gas with a temperature T_0.

The critical temperature of the hot-air screen ($y = y_*$) is determined by the condition that readily follows from (8):

$$\frac{4y_*}{(1+y_*)^2}\sqrt{\frac{1+A}{1+2A/(y_*(1+y_*))}} = 1. \tag{10}$$

The solutions of Eq. (10) for several values of A are listed in Table 1.

For screen temperatures lower than the critical one, the SW intensity is higher than that of the wave propagating in an isothermal gas with a constant temperature T_0. The maximum increase in the relative excess pressure at the wave front behind the screen and the corresponding temperature in it $T_m(y_m)$ can be found using Eq. (8). Equating the derivative with respect to y to zero, one can easily obtain the equation

$$y^3_m - y_m - 3A = 0, \tag{11}$$

which has the analytical solution

$$y_m = \begin{cases} \sqrt[3]{3A/2+\sqrt{9A^2/4-1/27}} + \sqrt[3]{3A/2-\sqrt{9A^2/4-1/27}}, & A \geqslant 2\sqrt{3}/27, \\ 2\sqrt{3}\cos\left[\arccos\left(9\sqrt{3}A/2\right)/3\right]/3, & A < 2\sqrt{3}/27. \end{cases} \tag{12}$$

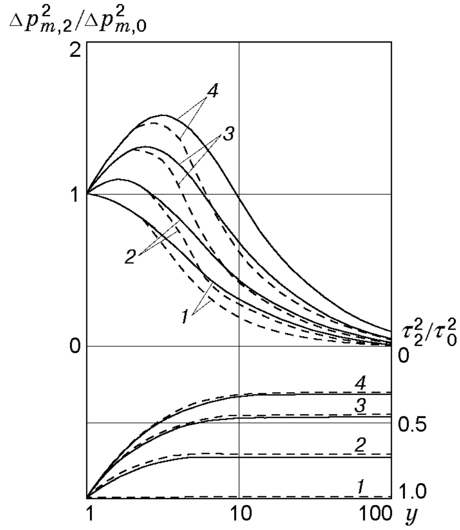

Fig. 1. Relative initial excess pressure at the wave front and relative duration of the compression stage behind the screen versus the parameter y for $A = 0$ (1), 1 (2), 4 (3), and 10 (4); the solid curves refer to the calculation for a constant ratio of specific heats and a constant molar mass; the dashed curves show the calculation with allowance for dissociation and ionization.

With (11), expression (8) for the maximum relative excess pressure can be brought to the form

$$\frac{\Delta p^2_{m,2}}{\Delta p^2_{m,0}}\Big|_m = \frac{4y_m}{(1+y_m)^2}\sqrt{\frac{3(1+A)}{1+2y_m}}. \tag{13}$$

The values of y_m and those of the maximum increase in the relative excess pressure at the SW front behind the screen predicted for several values of A by Eqs. (12) and (13) are listed in Table 2. It follows from this table that, as the parameter A increases (for a weak SW via increasing screen thickness), the maximum increase in the relative excess pressure at the wave front also increases. As $A \to \infty$, it follows from (13), in view of (12), that $\Delta p^2_{m,2}/\Delta p^2_{m,0}|_m \to 2\sqrt{2} \simeq 2.83$, i.e., the excess pressure in the wave behind the screen may be three times the excess pressure in the wave in an isothermal gas with a temperature T_0. It can be easily shown that the pressure in the wave behind the screen is always lower that the pressure in the wave approaching the screen; i.e., the wave-front pressure does not increase as the wave traverses the screen, which is also the case of cumulation of shock waves in layered condensed systems [7].

The solid curves in Fig. 1 show dependences (8) and (9) for several values of A. These dependences allow one to estimate the relative parameters for waves leaving hot-air screens with different temperatures. Curve 1 ($A = 0$), which shows the data for an infinitesimally weak wave ($\Delta p^1_{m,0} \to 0$), coincides with the solution obtained in the acoustic approximation (2).

Further evolution of parameters of the wave propagating in medium 2 behind the screen is described by the following relations, which are similar to (3):

$$\Delta p_{m,2} = \Delta p^2_{m,2} \Big/ \sqrt{1 + \frac{k+1}{2k}\,\frac{\Delta p^2_{m,2}}{p_0}\,\frac{x-x_2}{\tau^2_2 c_0}}, \quad \tau_2 = \tau^2_2\sqrt{1 + \frac{k+1}{2k}\,\frac{\Delta p^2_{m,2}}{p_0}\,\frac{x-x_2}{\tau^2_2 c_0}}.$$

In view of (7), we have

$$\Delta p_{m,2} = \Delta p^1_{m,0}\,\frac{4y}{(1+y)^2} \Big/ \sqrt{1 + \frac{k+1}{2k}\,\frac{\Delta p^1_{m,0}}{p_0}\Big(\frac{2}{y(1+y)}\,\frac{h}{\tau^1_0 c_0} + \frac{4y}{(1+y)^2}\,\frac{x-x_2}{\tau^1_0 c_0}\Big)},$$

$$\tau_2 = \tau^1_0\sqrt{1 + \frac{k+1}{2k}\,\frac{\Delta p^1_{m,0}}{p_0}\Big(\frac{2}{y(1+y)}\,\frac{h}{\tau^1_0 c_0} + \frac{4y}{(1+y)^2}\,\frac{x-x_2}{\tau^1_0 c_0}\Big)}.$$

Assuming $x = x_1$ to be the initial section for an SW propagating in an isothermal gas with a temperature T_0, we obtain the relations

$$\frac{\Delta p_{m,2}}{\Delta p_{m,0}} = \frac{4y}{(1+y)^2} \sqrt{1 + \frac{k+1}{2k} \frac{\Delta p^1_{m,0}}{p_0} \frac{x-x_1}{\tau^1_0 c_0}} \bigg/ \sqrt{1 + \frac{k+1}{2k} \frac{\Delta p^1_{m,0}}{p_0} \Big(\frac{2}{y(1+y)} \frac{h}{\tau^1_0 c_0} + \frac{4y}{(1+y)^2} \frac{x-x_2}{\tau^1_0 c_0} \Big)},$$

$$\frac{\tau_2}{\tau_0} = \sqrt{1 + \frac{k+1}{2k} \frac{\Delta p^1_{m,0}}{p_0} \Big(\frac{2}{y(1+y)} \frac{h}{\tau^1_0 c_0} + \frac{4y}{(1+y)^2} \frac{x-x_2}{\tau^1_0 c_0} \Big)} \bigg/ \sqrt{1 + \frac{k+1}{2k} \frac{\Delta p^1_{m,0}}{p_0} \frac{x-x_1}{\tau^1_0 c_0}},$$

which, on introducing the parameter A, may be recast as

$$\frac{\Delta p_{m,2}}{\Delta p_{m,0}} = \frac{4y}{(1+y)^2} \sqrt{\frac{1+A}{1+2A/(y(1+y))}} \sqrt{1 + \frac{1}{1+1/A} \frac{x-x_2}{h}} \bigg/ \sqrt{1 + \frac{4y}{(1+y)^2} \frac{1}{2/(y(1+y))+1/A} \frac{x-x_2}{h}}; \qquad (14)$$

$$\frac{\tau_2}{\tau_0} = \sqrt{\frac{1+2A/(y(1+y))}{1+A}} \sqrt{1 + \frac{4y}{(1+y)^2} \frac{1}{2/(y(1+y))+1/A} \frac{x-x_2}{h}} \bigg/ \sqrt{1 + \frac{1}{1+1/A} \frac{x-x_2}{h}}. \qquad (15)$$

It follows from Eqs. (14) and (15) that, depending on the relation of coefficients at the arguments in the numerator and denominator, the relative excess pressure at the wave front and the duration of the compression stage in the wave behind the screen may either increase or decrease. For

$$\frac{1}{1+1/A} < \frac{4y}{(1+y)^2} \Big(\frac{2}{y(1+y)} + \frac{1}{A} \Big)^{-1} \quad \text{or} \quad A > \frac{y}{2} \frac{(y-1)^2}{2y^2-y-1}, \qquad (16)$$

the relative excess pressure decreases, while the duration of the compression stage increases from the initial values (8) and (9). At infinity, both parameters tend to the same limiting value

$$\Delta p_{m,2}/\Delta p_{m,0}\Big|_\infty = \tau_2/\tau_0\Big|_\infty = 2\sqrt{y}/(1+y). \qquad (17)$$

Since $y > 1$, the limiting values of the relative wave parameters behind the screen are always lower than unity.

As an example, we consider the case $y = 2$. For $A = 2$, the relative pressure at the wave front behind the screen attains its highest value (see Table 2) $\Delta p^2_{m,2}/\Delta p^2_{m,0} = 1.1926$, and according to (9), the duration of the compression stage is $\tau^2_2/\tau^2_0 = 0.5556$. Away from the screen, according to (17), we have $\Delta p_{m,2}/\Delta p_{m,0} = \tau_2/\tau_0 \to 0.9428$, i.e., the relative excess pressure at the wave front decreases, whereas the duration of the compression stage becomes longer. Equating relation (14) to unity, we can easily estimate the distance at which the pressure at the wave front behind the screen becomes equal to the pressure in the wave traveling in an isothermal gas with a temperature T_0: $x - x_2 = 3.5625h$. According to (16), the regime of wave propagation behind the screen changes at $A = 0.2$. For instance, for $A = 0.1$, according to (8) and (9), we have $\Delta p^2_{m,2}/\Delta p^2_{m,0} = 0.9171$ and $\tau^2_2/\tau^2_0 = 0.9692$. At infinity, the relative parameters remain unchanged, both being equal to 0.9428, i.e., the relative excess pressure in the wave increases, while the duration of the compression stage decreases.

In a similar manner, we can consider the passage of the incident wave through one or several additional hot-gas screens, using relations (14) and (15) for determining the wave parameters.

Figure 2 shows the numerical solutions of the problem of a low-intensity plane shock wave that traverses, one by one, two hot-air screens ($k = 1.4$) with temperatures $T_1 = 805.6$ K ($y = 1.672$). These solutions were obtained using an algorithm that involved the calculation of the wave-front parameters and contact discontinuities by the method of characteristics and the solution of the Riemann problem on each contact surface [5]. In the initial section $x_0 = 1$ m, a wave with a triangular profile and parameters $\Delta p^0_{m,0}/p_0 = 0.15$ and $\tau^0_0 = 0.005$ sec was set. The first hot-air screen was located between the sections $x_1 = 6$ m and $x_2 = 24.2$ m, and the second one between the sections $x_3 = 28$ m and $x_4 = 60$ m. For both screens, $A = 1$, i.e., the relative excess pressures at the wave front leaving each of the two screens were assumed to be maximum possible (see Table 2).

Curve 1 in Fig. 2 shows the wave-front pressure in a wave traveling in isothermal air with a temperature $T_0 = 288.16$ K, and curves 2 and 3 show the pressure behind the first and second screens, respectively. A comparison of the numerical and analytical solutions shows them to differ only within the inaccuracy typical of the theory of short waves for the indicated SW intensity. For instance, the numerical solution predicts the values $\Delta p^2_{m,2}/p_0 = 0.09998$ and $\Delta p^4_{m,4}/p_0 = 0.07309$ for the initial excess pressure behind the first and second screens, respectively, whereas the analytical relations yield $\Delta p^2_{m,2}/p_0 = 0.09952$ and $\Delta p^4_{m,4}/p_0 = 0.07271$.

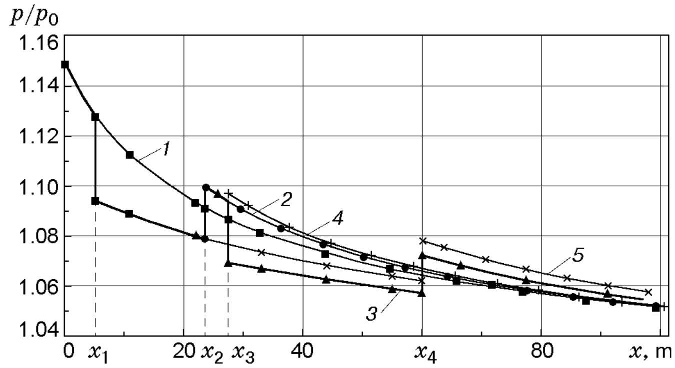

Fig. 2. Pressure variation at the SW front versus the distance behind one screen (2 and 3) and two screens (4 and 5) of thickness h and temperature 805.6 K: curve 1 shows the values for a wave traveling in air with $T_0 = 288.16$ K; curve 2 refers to $h = x_2 - x_1$, curve 3 to $h = (x_2 - x_1) + (x_4 - x_3)$, curve 4 to $h = x_3 - x_1$, and curve 5 to $h = x_4 - x_1$.

Since the specific impulse of the compression stage in a triangular SW is $i = \Delta p_m \tau/2$, it follows from (14) and (15) that

$$i_2 = 4yi_0/(1+y)^2, \tag{18}$$

i.e., the impulse due to the excess pressure in the wave leaving the hot-air screen at an arbitrary temperature ($y > 1$) is smaller than that in the wave traveling in isothermal air. It should be noted that relation (18) holds only for the first SW leaving the screen. In the situation under study, a reflected SW is formed in the screen, which subsequently propagates back from the inlet section of the screen and transfers an additional impulse to the gas behind it. This process recurs over and over, and the total impulse due to the excess pressure in the wave train that passed the screen becomes equal to the impulse of the SW traveling in isothermal air. For instance, in the above numerical example, the impulse of the SW traveling in isothermal air is $i_0 = 35.93$ Pa $\cdot$ sec at the section $x = 25$ m, whereas the impulse of the first wave leaving the screen is $i_2 = 33.36$ Pa $\cdot$ sec, i.e., $i_2 = 0.928 i_0$ [relation (18) yields the impulse ratio of 0.936]. The total impulse of the first three waves behind the hot-air screen at the above-indicated distance equals 35.57 Pa $\cdot$ sec, i.e., it coincides with i_0 within 1%.

Curves 4 and 5 in Fig. 2 show the pressure at the wave front as it leaves screens with identical temperatures $T_1 = 805.6$ K and with thicknesses $x_3 - x_1$ and $x_4 - x_1$, respectively. It follows from Fig. 2 that an increase in the screen thickness even without any change in the screen temperature results in a pressure increase in the gas behind the screen.

At high temperatures, the ratio of specific heats and the molar mass of air become different owing to ionization and dissociation processes. In this case, the properties of air can be described using caloric and thermal equations of state constituting perfect-gas equations with an effective ratio of specific heats $k_{\rm eff}$ and an effective molar mass $\mu_{\rm eff}$ [8]:

$$k_{\rm eff} = \begin{cases} k^* - 0.042(\varepsilon/\varepsilon^*)^2, & \varepsilon \leqslant \varepsilon^*, \\ a^* + (1.36 - a^*)\exp[0.223(1 - \varepsilon/\varepsilon^*)], & \varepsilon > \varepsilon^*, \end{cases}$$
$$\mu_{\rm eff} = \begin{cases} 28.96, & \varepsilon \leqslant \varepsilon^*, \\ 11.5 + 17.46\exp[0.0445(1 - \varepsilon/\varepsilon^*)], & \varepsilon > \varepsilon^*. \end{cases} \tag{19}$$

Here ε is the specific internal energy of the gas, $k^* = 1.402$, $\varepsilon^* = 1.116 \cdot 10^6$ J/kg, $a^* = 1 + 0.163/[1 - 0.0573\ln(\rho/\rho^*)]$, and $\rho^* = 1.2921$ kg/m^3.

Figure 3 shows the effective ratio of specific heats $k_{\rm eff}$ and the effective molar mass $\mu_{\rm eff}$ of air as functions of the parameter y under a constant initial pressure $p_0 = 0.1013$ MPa. These dependences were calculated by relations (19). In the range of y considered, the ratio of specific heats varies from 1.4 to 1.11, and the molar mass from 28.96 and 11.5, i.e., the change is substantial enough to affect the protective properties of the hot-air screen.

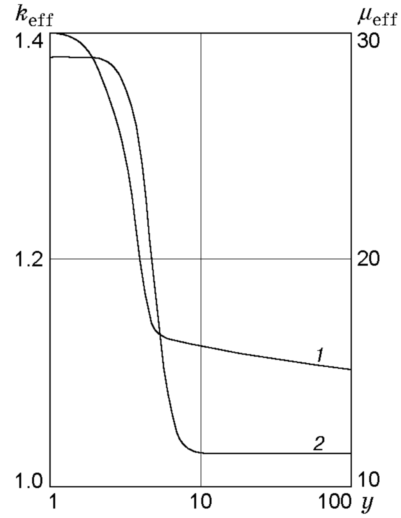

Fig. 3. Effective ratio of specific heats k_{eff} (1) and effective molar mass μ_{eff} (2) of air versus the parameter y.

For a weak plane SW with a triangular profile behind a hot-air screen with parameters T_1, k_1 and μ_1, one can easily derive relations for the relative initial excess pressure and for the duration of the compression stage:

$$\frac{\Delta p_{m,2}^2}{\Delta p_{m,0}^2} = \frac{4zy}{(1+zy)^2}\sqrt{1+A}\Bigg/\sqrt{1+\frac{k_1+1}{k_0+1}\Big(\frac{k_0}{k_1}\Big)^2\frac{2}{zy(1+zy)}\,A}\,; \tag{20}$$

$$\frac{\tau_2^2}{\tau_0^2} = \sqrt{1+\frac{k_1+1}{k_0+1}\Big(\frac{k_0}{k_1}\Big)^2\frac{2}{zy(1+zy)}A}\sqrt{\frac{1}{1+A}}\,. \tag{21}$$

Here $z = \sqrt{k_0\mu_0/(k_1\mu_1)}$.

Relations (20) and (21) are similar to (8) and (9) for the same gas with a constant ratio of specific heats and a constant molar mass; they differ only by a new argument zy and by an additional coefficient; the above analysis, therefore, is also applicable on the whole to a more complex case of a hot-air screen with variable k and μ. The dashed curves in Fig. 1 show the relative initial excess pressure $\Delta p_{m,2}^2/\Delta p_{m,0}^2$ at the front of a wave behind the hot-air screen and the duration of the compression stage τ_2^2/τ_0^2 as functions of the parameter y, both obtained by relations (19).

At high temperatures, a change in the air composition only weakly affects the relative duration of SW passage across the screen. The curves representing relations (9) and (21) originate at one point ($\tau_2^2/\tau_0^2 = 1$) at $y = 1$ and have one asymptote $1/\sqrt{1+A}$ as $y \to \infty$. The largest difference between the curves is observed for $\log y \approx 0.8$; it increases with A reaching to 7% at $A = 10$.

For identical values of A, the relative initial pressure at the SW front behind the hot-air screen is always lower than that in air with constant k and μ. As a result, hot air starts displaying its protective properties at lower temperatures, whereas identical temperatures, the decrease in SW pressure is more intense as compared to the case of air with constant properties. For instance, for $A = 4$, the critical value of the parameter y_* in air is approximately 26% lower ($y_* \approx 4.491$) compared to air with constant k and μ, while for $y = 10$, the relative excess pressure at the wave front behind the screen is higher approximately by 27%.

The above results are valid for low-intensity shock waves that obey the theory of short waves. As is shown above, with increasing pressure at the SW front, the errors inherent to this theory rapidly increase, becoming too high for an adequate analysis already at $\Delta p_m/p_0 = 0.5$, and the phenomenon of interest can be treated only numerically. For finite-intensity waves, the numerical algorithm of [5] was used. The solution of the problem for a shock wave with a triangular profile (with parameters $\Delta p_{m,0}^0/p_0 = 1.36$ and $\tau_0^0 = 0.00861$ sec in the initial section $x_0 = 1$ m) traversing a hot-air screen between the sections $x_1 = 6$ m and $x_2 = 24.2$ m with temperature a

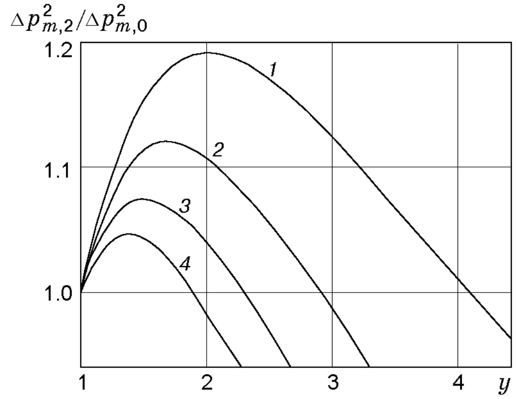

Fig. 4

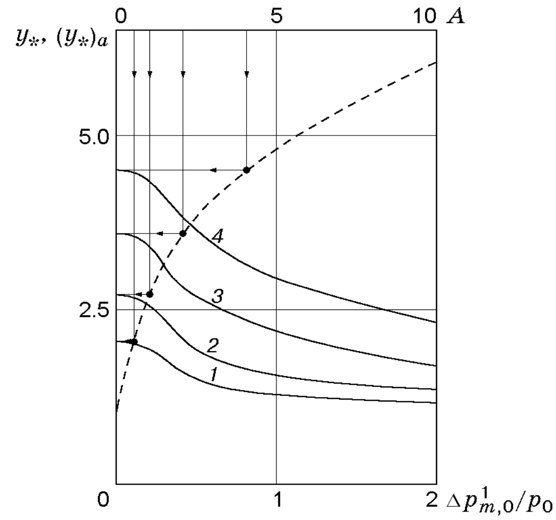

Fig. 5

Fig. 4. Relative initial excess pressure at the SW front behind hot-air screens versus the parameter y for $\Delta p^1_{m,0}/p_0 \to 0$ (1) and $\Delta p^1_{m,0}/p_0 = 0.4305$ (2), 0.8331 (3), and 1.5387 (4).

Fig. 5. Critical value of y_* as a function of the incident SW intensity in air (solid curves) and the dependence $(y_*)_a(A)$ (dashed curve) for $A = 0.5$ (1), 1 (2), 2 (3), and 4 (4).

$T_1 = 805.6$ K ($y = 1.672$) and constant ratio of specific heats $k = 1.4$ shows that the relative excess pressure at the wave front immediately behind the screen increases approximately by 18%, i.e., almost twofold compared to the case of a low-intensity wave ($\Delta p^0_{m,0}/p_0 = 0.15$). Nevertheless, this does not mean that the temperature range where the relative excess pressure and its highest value behind the screen grow becomes more extended as the wave intensity increases, because the values of A in the examples considered were different. We have $A = 1$ for a weak shock wave and $A = 4.585$ for the approaching wave under consideration (with parameters $\Delta p^1_{m,0}/p_0 = 1$ and $\tau^1_0 = 0.01$ sec); an increase in the screen thickness and, hence, in the parameter A at a constant temperature T_1, as was shown above (see Fig. 2), increases the relative pressure at the wave front behind the hot-air screen. In addition, as the finite-intensity wave propagates from the initial section to the screen boundary, its profile changes and becomes convex relative to the time axis.

To analyze the effect of the incident SW intensity on the protective properties of hot-air screens, we numerically solved the problem about a wave (with parameters $\Delta p^0_{m,0}/p_0 = 0.5$, 1.0, and 2.0, and $\tau^0_0 = 0.01$ sec in the initial section $x_0 = 1$ m) traversing hot-air screens ($k = 1.4$) with different temperatures. The law of pressure variation in the initial section $\Delta p^0_0(t)$ was chosen such that the wave had a triangular profile in front of the screen (at $x_1 = 6$ m), and the wave parameters for the above-indicated three cases were: 1) $\Delta p^1_{m,0}/p_0 = 0.4305$ and $\tau^1_0 = 0.01087$ sec; 2) $\Delta p^1_{m,0}/p_0 = 0.8331$ and $\tau^1_0 = 0.01061$ sec; 3) $\Delta p^1_{m,0}/p_0 = 1.5387$ and $\tau^1_0 = 0.01034$ sec. The screen thickness was 20.0, 10.1, and 5.3 m, respectively, and we assumed that $A = 2$ in all three cases.

Figure 4 shows the initial parts of the dependences of the relative excess pressure at the wave front behind the screen on the parameter y for the above-indicated cases (curves 2–4) and for a low-intensity wave (curve 1). As the incident SW intensity increases, the critical temperature of the hot-air screen decreases (approximately by a factor of 4.7 for $\Delta p^2_{m,2}/p_0 = 1.5387$), and the relative excess pressure at the wave front behind the screen decreases even more appreciably, i.e., the protective properties of the screen improve.

The main characteristic of the protective properties of a hot-air screen is the critical temperature T_*, on exceeding which the screen starts displaying its protective properties. For a low-intensity air wave, the critical values of $(y_*)_a$ predicted by relation (20) with allowance for (19) are listed in Table 1. In the case of a finite-intensity SW, the critical temperature of the screen (the parameter y_*) can be found numerically. We solved the problem for the three above-indicated intensities of an incident SW with a triangular profile and for hot-air screens with varied properties, whose thickness was chosen such that to ensure constancy of the parameter A.

The solid curves in Fig. 5 show the critical values of y_* as functions of the incident SW intensity for several values of A; the dashed curve in the same figure shows the dependence $y_*(A)$ for a low-intensity wave (see Table 1).

It follows from Fig. 5 that, as the intensity of the incident SW increases, the critical temperature of the screen (y_*) decreases abruptly, the shapes of the dependences for different values of A being similar. Close inspection of the data obtained shows that the effect of the intensity of the incident SW with a triangular profile on the critical temperature of the hot-air screen may be generalized by the relation

$$y_* = 1 + \frac{(y_*)_a - 1}{1 + (\Delta p^1_{m,0}/(a p_0))^\alpha}, \tag{22}$$

where $(y_*)_a = \sqrt{(1 + 6.4A)/(1 + 0.046A^{1.25})}$, $a = 0.38 + 0.21A$, and $\alpha = 1.2$.

For $\Delta p^1_{m,0}/p_0 < 1$, the data obtained by formula (22) differ from numerical result by no more than 3%, and the difference may reach 10% for $\Delta p^1_{m,0}/p_0 \simeq 1.5$ (for small A), which allows a first-order estimation of the minimum temperature of the screen on exceeding which the screen starts displaying its protective properties.

REFERENCES

1. R. A. Prosser, "Possibility of mitigating the destructive effects of a thermonuclear explosion by means of materials which absorb light," *J. Appl. Phys.*, **58**, No. 5, 2083–2086 (1985).
2. B. V. Zamyshlyaev and Yu. S. Yakovlev, *Dynamic Loads due to Underwater Explosions* [in Russian], Sudostroenie, Leningrad (1967).
3. V. A. Andrushchenko, M. V. Meshcheryakov, and L. A. Chudov, "Application of finite-difference methods to the prediction of the interaction between shock waves with a thermal screen," *Mat. Model.*, **2**, No. 1, 49–55 (1990).
4. O. M. Belotserkovskii, V. A. Andrushchenko, and Yu. D. Shevelev, *Dynamics of Three-Dimensional Vortex Flows in Spatially Nonuniform Atmosphere: Numerical Experiment* [in Russian], Yanus-K, Moscow (2000).
5. S. S. Men'shakov and V. N. Okhitin, "Passage of an air shock wave through a heated region," *Oboron. Tekh.*, Nos. 1/2, 30–33 (2001).
6. L. V. Ovsyannikov, *Lectures on Gas-Dynamics Foundations* [in Russian], Nauka, Moscow (1981).
7. E. I. Zababakhin, "Shock waves in layered systems," *Zh. Éksp. Teor. Fiz.*, **49**, No. 2, 642–646 (1965).
8. V. V. Koren'kov and V. N. Okhitin, "Numerical determination of effect of explosive density on parameters of air shock waves," *J. Appl. Mech. Tech. Phys.*, No. 3, 403–406 (1983).